

Dispersion relations at finite temperature and density for nucleons and pions

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Abstract

We calculate the nucleonic and pionic dispersion relations at finite temperature (T) and non-vanishing chemical potentials (μ_f) in the context of an effective chiral theory that describes the strong and electromagnetic interactions for nucleons and pions. The dispersion relations are calculated in the broken chiral symmetry phase, where the nucleons are massive and pions are taken as massless. The calculation is performed at lowest order in the energy expansion, working in the framework of the real time formalism of thermal field theory in the Feynman gauge. These one-loop dispersion relations are obtained at leading order with respect to T and μ_f . We also evaluate the effective masses of the quasi-nucleon and quasi-pion excitations in thermal and chemical conditions as the ones of a neutron star.

Keywords: Chiral Lagrangians, Dispersion Relations, Finite Temperature, Chemical Potentials, Nucleons, Pions.

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1 Introduction

Effective chiral theories have become a major conceptual and analytical tool in particle physics driven by the need of a theory to describe the low-energy phenomenology of QCD. The foundations were formulated originally by Weinberg [1] to characterise the most general S-matrix elements for soft pion interactions and later it was further developed by Gasser and Leutwyler [2]. Effective chiral theories have shown to be an adequate framework to treat low-energy phenomenology [3]-[6], as they reproduce, at lowest order in the chiral expansion, the most important results from current algebras including the low-energy theorems, and at next-to-leading order, they give precise corrections to these results [3]. They have been widely applied to different problems as meson-meson, meson-baryon, photon-photon, photon-meson and photon-baryon scattering, photoproduction processes and rare kaon decays [7, 18].

The propagation properties of relativistic particles in plasmas at finite temperature is also a subject of increasing interest. It is well known that the interaction of a particle with a plasma in thermal equilibrium at temperature T modifies the Dispersion Relations (DR) with respect to the zero temperature situation. This phenomenon has been extensively investigated for the non-dense plasma case [19]-[30], i.e. when the chemical potential (μ_f) associated to the fermions of the thermal plasma is equal to zero: $\mu_f = 0$ and $T \neq 0$. In this case the Fermionic Dispersion Relations (FDR) have been studied for massless fermions in [19]-[22] and massive fermions in [23]-[30]. The FDR describe the propagation of the fermionic excitations of the plasma (quasi-fermions and quasi-holes) through the thermal background. These excitations are originated in the collective behaviour of the plasma system at low momentum.

On the other hand, DR describing the propagation of the fermionic excitations of a dense plasma at finite temperature can be found in literature [31]-[35]. For the dense plasma case at finite temperature, i.e. $\mu_f \neq 0$ and $T \neq 0$, the FDR have been calculated both for massless fermions in [31]-[34] and for massive fermions in [35]. These FDR have been calculated in the context of realistic physical models, as for instance, the Minimal Standard Model [29, 34].

In the present work we calculate the DR for quasi-nucleons and quasi-pions propagating in a plasma at finite temperature and non-vanishing chemical potentials. The calculation is performed for a $SU(2)_L \times SU(2)_R$ effective chiral Lagrangian with the chiral symmetry broken into $SU(2)_{L+R}$. This Lagrangian, which we introduce in section 2, describe the strong and electromagnetic interactions of massive nucleons and massless pions. The calculation is performed using the real time formalism of the thermal field theory [36]-[38] in the Feynman gauge. The one-loop DR are calculated at lowest order in the energy expansion and obtained taking the T^2 and μ_f^2 terms from the self-energy, as shown in section 3. As an application of the DR obtained, we evaluate the effective masses of the quasi-nucleon and quasi-pion excitations taking

the following values: $T = 150$ MeV, $\mu_p = 100$ MeV and $\mu_n = 2\mu_p$, being $\mu_p(\mu_n)$ the chemical potential for protons (neutrons) [43]. This evaluation is shown in section 4, as well as the discussion of the main results and conclusions.

2 Effective chiral Lagrangian at leading order in the energy expansion

Effective chiral theories are founded in the existence of an energy scale Λ_χ at which chiral symmetry $SU(N_f)_L \times SU(N_f)_R$, with N_f the number of flavours, breaks into $SU(N_f)_{L+R}$ leading to $N_f^2 - 1$ Goldstone bosons associated to the N_f broken generators. These Goldstone bosons are identified with the meson ground state octet for $N_f = 3$, and with the triplet of pions [2, 6] in the case of $N_f = 2$. The chiral symmetry of the Lagrangian is broken through the introduction of an explicit mass term for the nucleons.

A general form for a Lagrangian with $SU(2)_{L+R}$ symmetry describing the strong and electromagnetic interactions for massive nucleons and massless pions is [39, 40]:

$$\mathcal{L} = \frac{F_\pi^2}{4} \text{Tr} [D_\mu \Sigma D^\mu \Sigma^\dagger] + \mathcal{L}_{\pi N} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (2.1)$$

where

$$\begin{aligned} \mathcal{L}_{\pi N} = & \bar{N} i \gamma^\mu \partial_\mu N - i e \bar{N} \gamma^\mu A_\mu \left(\frac{1 + \tau_3}{2} \right) N + M \bar{N} N \\ & + i M g_A \bar{N} \gamma_5 \frac{\tau \cdot \pi}{2 F_\pi} N + M g_A^2 \bar{N} \left(\frac{\tau \cdot \pi}{2 F_\pi} \right)^2 N + \dots, \end{aligned} \quad (2.2)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (2.3)$$

$$\Sigma = e^{i \frac{\tau \cdot \pi}{F_\pi}}, \quad (2.4)$$

where the covariant derivative and electromagnetic charge are defined as

$$D_\mu \Sigma = \partial_\mu \Sigma + i e A_\mu [Q, \Sigma], \quad (2.5)$$

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix}. \quad (2.6)$$

Here π , N and A_μ represent the pion, nucleon and electromagnetic fields, $F_\pi = 93$ MeV is the pion decay constant, e is the electromagnetic coupling constant, $g_A = 1.26$ is the axial coupling constant, and M is the average nucleon mass.

3 Dispersion relations for nucleons and pions

In this section we calculate the DR for nucleons and pions in the framework of the Lagrangian given by (2. 1). We consider the propagation of the nucleonic and pionic excitations in a dense thermal plasma constituted by protons, neutrons, charged pions, neutral pions and photons, being this plasma characterised by $\mu_{f_i} \neq 0$, where f_i represents the different fermion species. The calculation is performed in the real time formalism of the thermal field theory in the Feynman gauge. The real part of the nucleonic and pionic self-energies are evaluated at lowest order in the energy expansion and at one-loop order $(g_A/F_\pi)^2$, considering only the leading contributions in T and μ_f .

The Feynman rules for the vertices at finite temperature and density (Fig. 1) are the same as those at $T = 0$ and $\mu_f = 0$, while the propagators in the Feynman gauge for photons $D_{\mu\nu}(p)$, pions $D(p)$ and massive nucleons $S(p)$ are [41]:

$$D_{\mu\nu}(p) = -g_{\mu\nu} \left[\frac{1}{p^2 + i\epsilon} - i\Gamma_b(p) \right], \quad (3. 1)$$

$$D(p) = \frac{1}{p^2 + i\epsilon} - i\Gamma_b(p), \quad (3. 2)$$

$$S(p) = \frac{\not{p}}{p^2 + m_f^2 + i\epsilon} + i\not{p}\Gamma_f(p), \quad (3. 3)$$

where p is the particle four-momentum and the plasma temperature T is introduced through the functions $\Gamma_b(p)$ and $\Gamma_f(p)$, which are given by

$$\Gamma_b(p) = 2\pi\delta(p^2)n_b(p), \quad (3. 4)$$

$$\Gamma_f(p) = 2\pi\delta(p^2)n_f(p), \quad (3. 5)$$

with

$$n_b(p) = \frac{1}{e^{(p \cdot u)/T} - 1}, \quad (3. 6)$$

$$n_f(p) = \theta(p \cdot u)n_f^-(p) + \theta(-p \cdot u)n_f^+(p), \quad (3. 7)$$

being $n_b(p)$ the Bose–Einstein distribution function, and the Fermi–Dirac distribution functions for fermions ($n_f^-(p)$) and anti-fermions ($n_f^+(p)$) are:

$$n_f^\mp(p) = \frac{1}{e^{(p \cdot u \mp \mu_f)/T} + 1}. \quad (3. 8)$$

In the distribution functions (3.6) and (3.7), u^α is the four–velocity of the CoM frame of the plasma, with $u^\alpha u_\alpha = 1$.

The broken chiral symmetry, with hadronic non zero densities, has been predicted to be restored at about the same temperature at which deconfinement sets in. This temperature is of the order of 200 MeV as it is indicated by lattice calculations [42].

3.1 Nucleonic Dispersion Relation

Using the Feynman diagrams given in Fig. (2), we calculate the FDR for quasi-protons and quasi-neutrons.

In order to apply a similar procedure to that followed in [21, 29, 34], we first consider the hypothetical case of massless nucleons. In this case, we obtain two solutions: one describing the propagation of quasi-fermions

$$w(k) = M_{p,n} + \frac{k}{3} + \frac{k^2}{3M_{p,n}} + \mathcal{O}(k^3), \quad (3.9)$$

and another one describing the propagation of quasi-holes

$$w(k) = M_{p,n} - \frac{k}{3} + \frac{k^2}{3M_{p,n}} + \mathcal{O}(k^3). \quad (3.10)$$

We observe that if $k = 0$, $w(k) = M_{p,n}$. Then $M_p(M_n)$ can be interpreted as the effective mass of the quasi-protons (quasi-neutrons), and their expressions are:

$$M_p^2 = \left(\frac{3g_A^2 M^2}{64F_\pi^2} + \frac{e^2}{8} \right) T^2 + \frac{g_A^2 M^2}{32\pi^2 F_\pi^2} \left(\mu_n^2 + \frac{\mu_p^2}{2} \right) + \frac{e^2 \mu_p^2}{8\pi^2} \quad (3.11)$$

and

$$M_n^2 = \frac{3g_A^2 M^2}{64F_\pi^2} T^2 + \frac{g_A^2 M^2}{32\pi^2 F_\pi^2} \left(\frac{\mu_n^2}{2} + \mu_p^2 \right). \quad (3.12)$$

For the limit $k \gg M_{p,n}$ the FDR are:

$$w(k) = k + \frac{M_{p,n}^2}{k} - \frac{M_{p,n}^4}{2k^3} \text{Log}\left(\frac{2k^2}{M_{p,n}^2}\right) + \dots \quad (3.13)$$

$$w(k) = k + 2ke^{-2k^2/M_{p,n}^2} + \dots \quad (3.14)$$

For very high momentum the relations (3.13) and (3.14) become the ordinary DR for a massless fermion propagating in the vacuum, i.e. $w(k) = k$.

It is easy to demonstrate that $\mu_f = \int_0^\infty dp [n_f^-(p) - n_f^+(p)]$. This result shows that the chemical potential is associated with the difference between the number of nucleons over anti-nucleons. The later means that if a dense thermal plasma is characterised by $\mu_f > 0$, then the plasma presents an excess of nucleons over anti-nucleons.

Now we consider the more realistic case where nucleons are massive. In this case the FDR are valid for $T < T_c$, where $T_c \sim 200$ MeV is the critical temperature

of the chiral phase transition in non-zero hadronic density [42]. We observe that $m_{p,n} > M_{p,n}$, where $m_p(m_n)$ is the rest mass of the proton (neutron) and $M_{p,n}$ are given by (3. 11) and (3. 12). In the limit $m_{p,n}^2 \gg M_{p,n}^2$ the FDR become [24]:

$$w(k)^2 = k^2 + m_{p,n}^2 + M_{p,n}^2. \quad (3. 15)$$

Starting from relation (3. 15) and equations (3. 11), (3. 12), we obtain a general expression for the nucleon effective mass splitting ΔM_N^2 :

$$\Delta M_N^2 = m_p^2 - m_n^2 + \frac{e^2 T^2}{8} + \frac{1}{8\pi^2} \left(\frac{g_A^2 M^2 \mu_n^2}{8F_\pi^2} - \frac{g_A^2 M^2 \mu_p^2}{8F_\pi^2} + e^2 \mu_p^2 \right). \quad (3. 16)$$

3.2 Pionic Dispersion Relation

Using the Feynman rules given in Fig. (1), we obtain the following DR for quasi-pions:

$$w(k)^2 = k^2 + M_{\pi^\pm, \pi^0}^2, \quad (3. 17)$$

where $M_{\pi^\pm}(M_{\pi^0})$ is the effective mass for charged (neutral) quasi-pions, and their expressions are:

$$M_{\pi^\pm}^2 = \frac{T^2}{12} \left(\frac{g_A^2 M^2}{F_\pi^2} + e^2 \right) + \frac{g_A^2 M^2}{8\pi^2 F_\pi^2} (\mu_n^2 + \mu_p^2) \quad (3. 18)$$

and

$$M_{\pi^0}^2 = \frac{g_A^2 M^2}{8\pi^2 F_\pi^2} \left(\frac{2\pi^2 T^2}{3} + \mu_n^2 + \mu_p^2 \right). \quad (3. 19)$$

From (3. 18) and (3. 19) we obtain the pion effective mass splitting ΔM_π^2 :

$$\Delta M_\pi^2 = \frac{e^2 T^2}{12}. \quad (3. 20)$$

4 Results and conclusions

We now give the results of the calculation for the effective masses of quasi-nucleons and quasi-pions. We have used the following values $m_p = 938.271$ MeV, $m_n = 939.566$ MeV, $M = 938.919$ MeV, $T = 150$ MeV, $\mu_p = 100$ MeV, $\mu_n = 200$ MeV, $e^2 = 0.095$. The temperature and chemical potential values are of the order of those in a neutron star [43]. The results for the effective masses are:

$$\begin{aligned} M_p &= 1036.5133 \text{ MeV} \\ M_n &= 1033.8394 \text{ MeV} \\ M_{\pi^\pm} &= 637.2312 \text{ MeV} \\ M_{\pi^0} &= 637.0914 \text{ MeV} \end{aligned}$$

where M_p , M_n , M_{π^\pm} and M_{π^0} are the effective masses for the proton, neutron, charged pions and the neutral pion, including the strong and electromagnetic interactions.

The effective mass splitting for nucleons and pions are:

$$\begin{aligned}\Delta(M_p - M_n) &= 2.6740 \text{ MeV} \\ \Delta(M_{\pi^\pm} - M_{\pi^0}) &= 0.1398 \text{ MeV}\end{aligned}$$

where $\Delta(M_{\pi^\pm} - M_{\pi^0})$ is due exclusively to the combined electromagnetic interaction and temperature effects, as shown at (3. 20). For the nucleons, from the total effective mass splitting $\Delta(M_p - M_n)$, the combined electromagnetic and temperature contribute is $\Delta_{em}(M_p - M_n) = 0.0058 \text{ MeV}$.

In conclusion, temperature effects enter into the effective mass splitting relations (3. 16) and (3. 20) exclusively in the electromagnetic interaction term, which at $T=0$ vanishes. Also, in the framework of our model we found that, for the chemical potentials and temperature used, the effective mass on the proton is bigger than the one of the neutron. Our results should be improved by considering massive pions and introducing the weak interaction, as well as using a realistic model for neutron stars, to be presented in short.

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Figure 1: Feynman Rules of the $\mathcal{L}_{\pi N}$.

Figure 2: Self-energy contributions for the calculation of FDR for: (a) Protons (b) Neutrons.

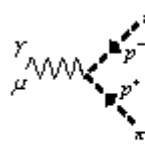
$$\begin{array}{c} \pi^+ \pi^- \\ \text{---} \text{---} \text{---} \\ p \quad n \quad p \end{array} + \begin{array}{c} \pi_0 \\ \text{---} \text{---} \text{---} \\ p \quad p \quad p \end{array} +$$

$$\begin{array}{c} \pi^+ \pi^- \pi_0 \\ \text{---} \text{---} \text{---} \\ p \quad p \end{array} + \begin{array}{c} \gamma \\ \text{---} \text{---} \text{---} \\ p \quad p \quad p \end{array}$$


(a)

$$\begin{array}{c} \pi^+ \pi^- \\ \text{---} \text{---} \text{---} \\ n. \quad p \quad n. \end{array} + \begin{array}{c} \pi_0 \\ \text{---} \text{---} \text{---} \\ n. \quad n. \quad n. \end{array} + \begin{array}{c} \pi^+ \pi^- \pi_0 \\ \text{---} \text{---} \text{---} \\ n. \quad n. \quad n. \end{array}$$

(b)



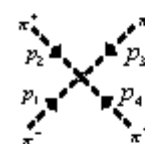
$$-ie(p^- - p^+)_{\mu}$$



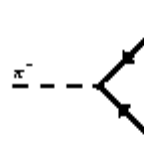
$$2ie^2 \varepsilon_{\mu\nu}$$




$$-ie\gamma_{\mu}$$



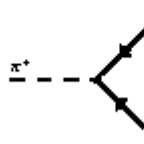
$$\frac{-i}{3F_{\pi}^2} \left[2(p_2 \cdot p_4 + p_1 \cdot p_3) + (p_1 + p_3)^2 \right]$$



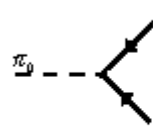
$$\frac{-ig_A M}{\sqrt{2}F_{\pi}} \gamma_5$$



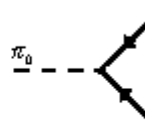
$$\frac{-i}{3F_{\pi}^2} \left[2(p^+ \cdot p^- + p_1 \cdot p_2) + (p_1 + p_2)^2 \right]$$



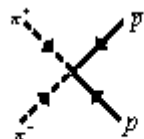
$$\frac{-ig_A M}{\sqrt{2}F_{\pi}} \gamma_5$$



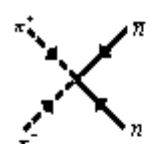
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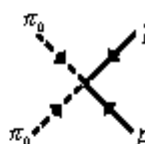
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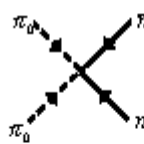
$$\frac{-ig_A^2 M}{4F_{\pi}^2}$$



$$\frac{-ig_A^2 M}{4F_{\pi}^2}$$



$$\frac{-ig_A^2 M}{8F_{\pi}^2}$$



$$\frac{-ig_A^2 M}{8F_{\pi}^2}$$